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Chan-Paton factors and higgsing from vacuum string field theory

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Abstract: We give a description of open strings stretched between N parallel D-branes in VSFT. We show how higgsing is generated as the branes are displaced: the shift in the mass formula for on-shell states stretched between different branes is due to a twist anomaly, a contribution localized at the midpoint.

Keywords: String Field Theory, Bosonic Strings, D-branes, Tachyon Condensation.

Contents

1. Introduction

Open String Field Theory (OSFT), [1], is a candidate for a non-perturbative definition of string theory. Central to it there is the concept of background independence: physics should be independent on the vacuum we decide to expand the theory on. Of course there can be vacua on which the theory might look simpler than in any other vacuum therefore, once such a vacuum is identified, the theory should take its simplest form around it. As the dynamical degrees of freedom of OSFT are open strings, a class of its vacua should contain all possible configurations of D-branes. Bosonic D-branes are unstable due to the omnipresence of the open string tachyon, hence they decay classically to a vacuum with no D-branes at all. The nature of this vacuum (tachyon vacuum) is universal, in the sense that it is independent of the details of the various BCFT's that describe the original configuration of branes. For this reason there is strong expectation that OSFT on the tachyon vacuum should be able to describe all open string physics and, possibly, the related closed string physics that arises non-perturbatively through the classical decay of the branes, see [2 – 6] for review and references therein. To overcome the difficulties in finding exact analytic solutions of OSFT, a model (Vacuum String Field Theory) has been conjectured by Rastelli Sen and Zwiebach which is a proposal for OSFT at the tachyon vacuum, [7]. This model highly simplifies the form of the OSFT action by replacing the usual BRST operator with a c-midpoint insertion. VSFT has passed many tests. Classical solutions relative to different branes have been obtained, [8 – 11] and the right ratios of tensions have been reproduced [12]. The small on shell perturbations of the solution relative to the D25-brane have been shown

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to cover the open string spectrum on a D25-brane [13 – 16]. Time dependent solutions interpolating from the D25-brane vacuum to the tachyon vacuum have been proposed, [17, 18]. However all these successes have to be contrasted with the singular nature of VSFT coming from the factorization of matter and ghost degrees of freedom. Hints on how such a singular nature arises have been given in [19, 20]: matter ghost factorization arises as a singular limit of a reparametrization of the worldsheet which shrinks the whole string to its midpoint. For this reason all of the classical solutions of VSFT exhibit a singular behavior at the midpoint and, as a consequence, observables are obtained by regularization of these singularities. Although a completely consistent regularization scheme has not been given yet (but see [21] for interesting developments) the leading matter-ghost factorized form of VSFT is very powerful and, in all the above-mentioned cases, it gives rise unambiguously to the right physics, once midpoint singularities are correctly regularized. In this note we give a simple description of open strings states living on a set of N D-branes. When the branes are coincident we encounter in the spectrum N^2 massless vectors, giving rise to a U(N) gauge symmetry. This symmetry is part of the huge gauge symmetry of VSFT when one considers matter-ghost factorized gauge transformations. The Chan Paton factors arise from particular combinations of left/right excitations on the sliver, that takes the form the generalized Laguerre polynomials discovered in [22], see also [6]. This U(N) structure is dynamically generated (it is an intrinsic part of a classical solution) and there is no need to add it by hand as in first quantized string theory or even in usual OSFT. In this sense background independence is manifest.

Using the translation operator $e^{ix\hat{p}}$ we construct an array of D24-branes and analyze its small on shell fluctuations. We show that open strings stretched between parallel branes at different positions are obtained by translating differently the left and right part of the classical solution. This is possible because the lump projector is left/right factorized. Of course this operation is ambiguous for what concerns the midpoint, since it does not have a left/right decomposition. Indeed we show that a naive use of left/right orthogonality cannot give rise to the correct shift in the mass formula, proportional to the *distance*^{2} between two D-branes. By using wedge-state regularization we show that in the sliver limit there is a non vanishing contribution which is completely localized at the midpoint and gives rise to the correct shift in the mass formula. The mechanism is that of a twist anomaly, [23], which has proven to be crucial for obtaining the spectrum of strings around a single D25-brane and to give the correct ratio of D-branes.

The note is organized as follows. After a brief review of the simplest classical solutions, the sliver and the lump, [8], in section 3 we review the construction of orthogonal Neumann (i.e. zero momentum) projectors using half string vectors and Laguerre polynomials, given in [22]. In the following section 4 we show how open string states with the correct Chan-Paton factors arise on a classical solution given by the sum of N such orthogonal projectors. In section 5 we consider a superposition of D24-branes and show how the transverse fluctuations are obtained, by simply exciting the transverse part of the classical solution with generic oscillators. Most of these are pure gauge but the excitation given by the midpoint oscillator cannot be gauged away. In section 6 we higgs the above system by translating the D24-branes at a certain distance from one another, thus creating an array in the transverse direction. We analyze the fluctuations of such a classical solution and show that the mass shift for strings stretched between different branes is generated by a twist anomaly. We further show that the wave-functional for such states is not continuous at the midpoint but gives rise to the expected change in the boundary conditions, living the midpoint position in the target space undetermined.

Some computations involving well known formulas for computing $*$-products on Neumann-type solutions are summarized in the appendix.

2. D-branes as projectors

The leading order VSFT action is

$$\mathcal{S}(\Psi) = -K\left(\frac{1}{2}\langle\Psi|\mathcal{Q}|\Psi\rangle + \frac{1}{3}\langle\Psi|\Psi * \Psi\rangle\right), \tag{2.1}$$

where

$$\mathcal{Q} = \frac{1}{2i}\left(c(i) - c(-i)\right) \tag{2.2}$$

given the purely ghost form of the kinetic operator, the classical solutions can be matter/ghost factorized

$$\Psi = \Psi_m \otimes \Psi_g \tag{2.3}$$

so the equation of motion factorizes too

$$\mathcal{Q}\Psi_g = -\Psi_g *_g \Psi_g \tag{2.4}$$

$$\Psi_m = \Psi_m *_m \Psi_m\,. \tag{2.5}$$

One can thus consider a universal ghost solution and concentrate in finding projectors of the matter star algebra. The operator definition of the $*_m$ product is

$$_{123}\langle V_3|\Psi_1\rangle_1|\Psi_2\rangle_2 =_3 \langle\Psi_1 *_m \Psi_2|\,, \tag{2.6}$$

where $\langle V_3|$ is the three string vertex, see [24 − 26, 6]. In the following we need both translationally invariant (D25-branes) and non-translationally invariant (Dk-branes) solutions. For simplicity we will consider the sliver and the lump, [8]. The former is translationally invariant and is defined by

$$|\Xi\rangle = \mathcal{N}e^{-\frac{1}{2}a^\dagger\cdot S\cdot a^\dagger}|0\rangle, \qquad a^\dagger\cdot S\cdot a^\dagger = \sum_{n,m=1}^{\infty} a_n^{\mu\dagger}S_{nm}a_m^{\nu\dagger}\eta_{\mu\nu}\,, \tag{2.7}$$

where $S = CT$ and

$$T = \frac{1}{2X}(1 + X - \sqrt{(1+3X)(1-X)})\,, \tag{2.8}$$

C is the twist matrix and $X = CV^{11}$ is the diagonal Neumann coefficient, see [24, 25, 6, 26] for details.

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To represent lower dimensional brane we need to break translation invariance in the transverse direction, this is usually done by passing to the oscillator basis

$$a_0^{(r)\alpha} = \frac{1}{2}\sqrt{b}\hat{p}^{(r)\alpha} - i\frac{1}{\sqrt{b}}\hat{x}^{(r)\alpha}, \qquad a_0^{(r)\alpha\dagger} = \frac{1}{2}\sqrt{b}\hat{p}^{(r)\alpha} + i\frac{1}{\sqrt{b}}\hat{x}^{(r)\alpha}, \tag{2.9}$$

where b is a free parameter and α, β denote transverse indices.

The transverse part of the vertex in this new basis becomes

$$|V_{3,\perp}\rangle' = K\, e^{-E'}|\Omega_b\rangle \tag{2.10}$$

with

$$K = \left(\frac{\sqrt{2\pi b^3}}{3(V_{00}+b/2)^2}\right)^{k/2}, \qquad E' = \frac{1}{2}\sum_{r,s=1}^{3}\sum_{M,N\geq 0} a_M^{(r)\alpha\dagger} V_{MN}'^{rs} a_N^{(s)\beta\dagger}\eta_{\alpha\beta} \tag{2.11}$$

where M,N denote the couple of indices $\{0,m\}$ and $\{0,n\}$, respectively. The coefficients $V_{MN}'^{rs}$ are given in [8, appendix B].

The lump solution $|\Xi_k'\rangle$ has the form (2.7) with S along the parallel directions and S replaced by S' along the perpendicular ones. In turn $S' = CT'$ and T' has the same form as T eq. (2.8) with X replaced by X'.

3. N coincident D25-branes

There are several ways to construct coincident branes solutions in VSFT, the one we are going to use is in terms of Laguerre polynomials, explicitly given in [22].

Consider a left string vector ξ_n^μ, such that

$$\rho_R\xi^\mu = 0 \tag{3.1}$$

$$\rho_L\xi^\mu = \xi^\mu\,. \tag{3.2}$$

The $\rho_{R,L}$ operators project into the the right/left Hilbert space at zero momentum, see [9].

With this half string vector it is possible to excite left-right symmetrically a string configuration, using the operator

$$\mathbf{x} = \left(a_\mu^\dagger, \xi^\mu\right)\left(a_\nu^\dagger, C\xi^\nu\right) = y\tilde{y}\,, \tag{3.3}$$

where $(\cdot,\cdot)$ means inner product in level space and the operators $\tilde{y}$ y are identified with right/left excitations. The half string vector ξ is normalized by the following condition and definition

$$\left(\xi_\mu, \frac{1}{1-T^2}\xi^\mu\right) = 1 \tag{3.4}$$

$$\left(\xi_\mu, \frac{T}{1-T^2}\xi^\mu\right) = -\kappa\,, \tag{3.5}$$

where $T = CS$ is the Sliver Neumann coefficient, (2.8).

For every choice of ξ satisfying 3.4, we can construct an infinite family of orthogonal projectors (D-branes) given by [22]

$$|\Lambda_n\rangle = (\kappa)^n\, L_n\left(\frac{\mathbf{x}}{\kappa}\right)|\Xi\rangle\,, \tag{3.6}$$

where $L_n(x)$ is the n-th Laguerre polynomial. These states obey the remarkable properties

$$|\Lambda_n\rangle * |\Lambda_m\rangle = \delta_{nm}|\Lambda_m\rangle \tag{3.7}$$
$$\langle\Lambda_n|\Lambda_m\rangle = \delta_{nm}\langle\Xi|\Xi\rangle\,. \tag{3.8}$$

Due to these properties, once the sliver is identified with a single D-brane, a stack of N D-branes can be given by

$$|N\rangle = \sum_{n=0}^{N-1}|\Lambda_n\rangle\,. \tag{3.9}$$

From (3.8) we further get that the *bpz* norm of such a solution is N-times the one of the sliver.

So far we have considered left-right symmetric projectors which are in one to one correspondence with type 0 Laguerre polynomial, there are however non left-right symmetric states corresponding to generalized Laguerre polynomials, they are given by, [6]

$$|\Lambda_{nm}\rangle = \sqrt{\frac{n!}{m!}}\kappa^m(iy)^{n-m}L_m^{n-m}\left(\frac{x}{\kappa}\right)|\Xi\rangle \qquad n\geq m \tag{3.10}$$
$$|\Lambda_{nm}\rangle = \sqrt{\frac{m!}{n!}}\kappa^n(i\tilde{y})^{n-m}L_n^{m-n}\left(\frac{x}{\kappa}\right)|\Xi\rangle \qquad m\geq n \tag{3.11}$$

and obey the properties[1]

$$|\Lambda_{nm}\rangle * |\Lambda_{pq}\rangle = \delta_{mp}|\Lambda_{nq}\rangle \tag{3.12}$$
$$\langle\Lambda_{nm}|\Lambda_{pq}\rangle = \delta_{mp}\delta_{nq}\langle\Xi|\Xi\rangle \tag{3.13}$$

note in particular that $|\Lambda_n\rangle = |\Lambda_{nn}\rangle$. With these states we can implement partial-isometry-like operations, see also [28]. Consider indeed

$$|-+\rangle = \sum_{n=0}^{N-1}|\Lambda_{n+1,n}\rangle \tag{3.14}$$
$$|+-\rangle = \sum_{n=0}^{N-1}|\Lambda_{n,n+1}\rangle\,. \tag{3.15}$$

It's trivial to see that

$$|+-\rangle * |-+\rangle = |N\rangle \tag{3.16}$$
$$|-+\rangle * |+-\rangle = |N\rangle - |\Xi\rangle\,. \tag{3.17}$$

[1] Another realization of this algebra is given in [27].

Note that any of the previous states can be obtained starting from the sliver by star products

$$|\Lambda_{nm}\rangle = (|-+\rangle)_*^n * |\Xi\rangle * (|+-\rangle)_*^m \,. \tag{3.18}$$

We have in particular

$$\begin{aligned} |+-\rangle * |\Xi\rangle &= 0 \\ |\Xi\rangle * |-+\rangle &= 0\,. \end{aligned} \tag{3.19}$$

As a final remark it is worth noting that the partial isometry that relates projectors to projectors is actually a $*$-rotation and hence a (matter ghost factorized) gauge transformation. We have indeed

$$\Lambda_{nn} = \mathrm{e}^{\frac{\pi}{2}(\Lambda_{nm}-\Lambda_{mn})}\,\Lambda_{mm}\,\mathrm{e}^{-\frac{\pi}{2}(\Lambda_{nm}-\Lambda_{mn})} \tag{3.20}$$

as can be easily checked from (3.12)

4. U(N) open strings

Let's recall that the (matter-ghost factorized) open string cohomology around a (matter-ghost factorized) classical solution $|\Psi\rangle$ is given by the following conditions

$$|\phi\rangle = |\phi\rangle * |\Psi\rangle + |\Psi\rangle * |\phi\rangle \tag{4.1}$$

$$|\phi\rangle \neq |\Lambda\rangle * |\Psi\rangle - |\Psi\rangle * |\Lambda\rangle\,. \tag{4.2}$$

The first representing $\mathcal{Q}_\Psi$-closed states while the second gauges away $\mathcal{Q}_\Psi$ exact-states.[2] In the case of N-coincident D25-branes the classical solution is given by (3.9).

As multiple D-branes are obtained starting from the sliver by multiple $*$-products via (3.18), a generic open string state on the sliver can acquire a Chan-Paton factor $(i,j) \in Adj[\mathrm{U}(N)]$ in the same way.

Let $|\{g\},p\rangle$ be an on-shell open string state on the sliver, identified by the collection of polarization tensors $\{g\}$ and momentum p. The Chan-Paton structure is simply given by

$$|(i,j);\{g\},p\rangle = (|-+\rangle)_*^i * |\{g\},p\rangle * (|+-\rangle)_*^j \,. \tag{4.3}$$

There is a subtlety here, related to twist anomaly, [23], and the consequent breakdown of $*$-associativity. Indeed the expression (4.3) is ambiguous in the overall normalization in front: it depends on how the various star products involved are nested. This is so because all the states we are considering are constructed on the sliver, which fails to satisfy unambiguously its equation of motion when states at non zero momentum enter the game. Consider for simplicity the Hata-Kawano tachyon state, [13, 29]

$$|p\rangle = \mathcal{N}\mathrm{e}^{(-ta^\dagger+ix)p}|\Xi\rangle = \mathcal{N}'\mathrm{e}^{ip\hat{X}\left(\frac{\pi}{2}\right)}|\Xi\rangle \tag{4.4}$$

$$t = 3\frac{T^2-T+1}{1+T}v_0 \tag{4.5}$$

[2]These conditions actually cover only the ghost-matter factorized cohomology

this state satisfies (weakly) the linearized equation of motion (LEOM) with the sliver state

$$|p\rangle * |\Xi\rangle \;=\; |\Xi\rangle * |p\rangle = \mathrm{e}^{-Gp^2}|p\rangle \tag{4.6}$$

$$G = \log 2 \quad \Rightarrow \quad p^2 = 1\,. \tag{4.7}$$

The quantity G gets a non vanishing value from the region very near $k = 0$ in the continuous basis, where some of the remarkable properties, encoding associativity, between Neumann coefficients breaks down due to singularities that are regulated in a non associative way (like level truncation). Indeed (4.6) violates associativity if, as is the case, $G \neq 0$

$$(|p\rangle * |\Xi\rangle) * |\Xi\rangle \neq |p\rangle * (|\Xi\rangle * |\Xi\rangle)\,. \tag{4.8}$$

Just to fix a convention (and stressing once more that the only ambiguity is in the overall normalization) we decide to do first all the star products at zero momentum (that do not develop twist anomaly) and, as the last operation, multiply the result with the state at definite momentum $|\{g\}, p\rangle$

Now we show that (4.3) satisfies the LEOM

$$|(i,j); \{g\}, p\rangle = |(i,j); \{g\}, p\rangle * |N\rangle + |N\rangle * |(i,j); \{g\}, p\rangle \tag{4.9}$$

using (3.19) we get the relations

$$|N\rangle * (|-+\rangle)_*^i \;=\; (|-+\rangle)_*^i * |N - i\rangle \tag{4.10}$$

$$(|+-\rangle)_*^j * |N\rangle \;=\; |N - j\rangle * (|+-\rangle)_*^j \tag{4.11}$$

which allow to write the LEOM as

$$|(i,j); \{g\}, p\rangle = (|-+\rangle)_*^i * \Big(|\{g\}, p\rangle * |N - j\rangle + |N - i\rangle * |\{g\}, p\rangle\Big) * (|+-\rangle)_*^j \tag{4.12}$$

It is proven in the appendix that

$$|\{g\}, p\rangle * |\Lambda_{n\geq 1}\rangle = |\Lambda_{n\geq 1}\rangle * |\{g\}, p\rangle = 0 \tag{4.13}$$

once the following conditions on the half string vector ξ^μ are satisfied

$$\left(t, \frac{1}{1 \pm T}\xi^\mu\right) = 0 \tag{4.14}$$

$$\left(g_{\mu\nu_1\ldots\nu_n}, \frac{1}{1 \pm T}\xi^\mu\right) = 0\,. \tag{4.15}$$

The first condition states that the half string vector ξ^μ should be "orthogonal" to the on-shell tachyon vector $t = 3\frac{T^2-T+1}{1+T}v_0$, this just constrains $2D$ components of ξ^μ out of $D\infty{-}1$, and as such it is easy to implement.[3] The second condition implies the "orthogonality" of the half string vector ξ^μ with any of the on-shell polarization tensors $\{g\}$. We know from

[3]These conditions are actually not needed if we represent the tachyon state as $\mathrm{e}^{ip\hat{X}(\frac{\pi}{2})}|\Xi\rangle$ since, up to overall normalizations, midpoint insertions commutes with the star product; the role of such conditions is to avoid extra terms when we use the CBH formula to pass to the oscillator expression $\mathrm{e}^{(-ta^\dagger+ix)p}|\Xi\rangle$.

previous works, [14, 16], that the level components of the polarization tensors should be identified with the $k = 0$ eigenvector(s), of the continuous basis at zero momentum, [30]. Again, in order for (4.15) to be satisfied, it is sufficient to ask that the components $\xi^\mu(k)$ vanish fast enough at $k = 0$, see [11, 16] for explicit realizations of this condition.

Given (4.13) it follows directly that

$$|\{g\},p\rangle * |N-j\rangle + |N-i\rangle * |\{g\},p\rangle = |\{g\},p\rangle * |\Xi\rangle + |\Xi\rangle * |\{g\},p\rangle \tag{4.16}$$

hence the LEOM simplifies to

$$|(i,j);\{g\},p\rangle = (|-+\rangle)_*^i * \Big(|\{g\},p\rangle * |\Xi\rangle + |\Xi\rangle * |\{g\},p\rangle\Big) * (|+-\rangle)_*^j\,. \tag{4.17}$$

This ensures the on-shellness of the state $|(i,j)\;;\;\{g\},p\rangle$ once this is true for $|\{g\},p\rangle$. We thus recover N^2 kinematical copies of the spectrum on a single D-brane. Note that the left/right structure of these states is the same as a $\mathrm{U}(N)$ double line notation, as the relations (3.12) certify. It should be noted that this Chan-Paton structure does not sit at the endpoints of the string, [31], but is rather "diluted" on the string halves. This can be traced back to the singular field redefinition that should relate OSFT with VSFT, see the conclusions.

5. N coincident D24-branes

A system of N coincident D24-branes can be represented by[4]

$$|N\rangle = \left(\sum_{n=0}^{N} |\Lambda_n\rangle\right) \otimes |\Xi'\rangle\,, \tag{5.1}$$

where the state $|\Xi'\rangle$ is the lump solution given in [8]. The open string string sector with Lorentz indices coming from the 25 dimensional world volume (N^2 tachyons, $\mathrm{U}(N)$-gluons, etc...) is exactly as in the previous section. In addition there are the physical states coming from transverse excitations. These states are given by exciting the transverse part of the classical solution $|\Xi'\rangle$ with oscillators. The Chan-Paton degrees of freedom are encoded in the worldvolume part of the state as in the previous section. For example the transverse scalars are given by[5]

$$|(ij);g^{25},p_\parallel\rangle = \mathcal{N}\,\mathrm{e}^{(-ta^\dagger+ix)p_\parallel}|\Lambda_{ij}\rangle \otimes g^{25}\cdot a_{25}'^\dagger|\Xi'\rangle\,. \tag{5.2}$$

It's easy to verify that these states satisfy the LEOM iff $p_\parallel^2 = 0$, we have indeed

$$\begin{aligned}|(ij);g^{25},p_\parallel\rangle &= |N\rangle * |(ij);g^{25},p_\parallel\rangle + |(ij);g^{25},p_\parallel\rangle * |N\rangle &(5.3)\\ &= 2^{-p_\parallel^2}\mathcal{N}\,\mathrm{e}^{(-ta^\dagger+ix)p_\parallel}|\Lambda_{ij}\rangle \otimes (\rho_L'+\rho_R')g^{25}\cdot a_{25}'^\dagger|\Xi'\rangle\,. &(5.4)\end{aligned}$$

The $\rho'_{L,R}$ are the left/right projectors with zero modes, see [9].

[4]Other possibilities, for example putting the Laguerre polynomials on the codimension, are related to this by partial isometry and hence, due to (3.20), should be gauge equivalent

[5]Note that once the relations (4.14) are implemented one can recast the Chan Paton indices directly on the classical solution and then act with oscillators to build onshell fluctuation

The level vector g^{25} is completely arbitrary, but only its midpoint part is not pure gauge. Indeed, exactly as in [32], we can try to gauge away any of the states (5.2)

$$\delta|(ij);g^{25},p_\parallel\rangle = |Q_{ij}\rangle * |N\rangle - |N\rangle * |Q_{ij}\rangle\,, \tag{5.5}$$

where

$$|Q_{ij}\rangle = -\mathrm{e}^{(-ta^\dagger+ix)p_\parallel}|\Lambda_{ij}\rangle \otimes u^{25}\cdot a'^\dagger_{25}|\Xi'\rangle\,. \tag{5.6}$$

We have

$$\delta|(ij);g^{25},p_\parallel\rangle = -|(ij);(\rho'_L-\rho'_R)u^{25},p_\parallel\rangle\,. \tag{5.7}$$

Thus the state is pure gauge if

$$g^{25} = (\rho'_L-\rho'_R)u^{25}\,. \tag{5.8}$$

It is well known that the operator $(\rho'_L-\rho'_R)$ just changes the twist parity of a given level vector. In the diagonal basis all vectors are paired except the one corresponding to $k=0$ that is only twist even, [33] (at least if we restrict ourselves to vectors that have a non vanishing overlap with Fock-space vectors, see [16]). Thus the gauge transformation (5.7) gauges away all the components of g^{25} except the $k=0$ one, which is the midpoint. One can construct higher transverse excitations by applying more and more transverse oscillators as in [14, 16]. Again only the $k=0$ oscillator(s) are not gauge trivial.

6. Higgsing

Now we want to "higgs" the previous system of N coincident D24-branes to an array of N D24-branes, displaced at a distance ℓ from one another in the transverse dimension y. This system is obtained by multiple translation of the previous classical solution (5.1).

$$|N^{(\ell)}\rangle = \sum_{n=0}^{N-1}\left(|\Lambda_n\rangle\otimes\mathrm{e}^{-in\ell\hat p}|\Xi'\rangle\right). \tag{6.1}$$

As in [8] it is very convenient to pass to the oscillator basis by

$$\hat p = \frac{1}{\sqrt b}\left(a_0+a_0^\dagger\right) \tag{6.2}$$

and to define the level vector

$$\beta_N = -\frac{i\ell}{\sqrt b}\left(1-T'\right)_{0N}\,. \tag{6.3}$$

The transverse part of the n-th D24-brane in (6.1) can thus be written as

$$|\Xi'_n\rangle = \mathrm{e}^{in\ell\hat p}|\Xi'\rangle = \mathrm{e}^{\frac{n^2}{2}\left(\beta,\frac{1}{1-T'}\beta\right)+n\beta\cdot a'^\dagger}|\Xi'\rangle\,. \tag{6.4}$$

As proven in [34] we have

$$|\Xi'_n\rangle * |\Xi'_m\rangle = \delta_{nm}|\Xi'_n\rangle \tag{6.5}$$

$$\langle\Xi'_n|\Xi'_m\rangle = \delta_{nm}\langle\Xi'|\Xi'\rangle\,. \tag{6.6}$$

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We recall here that the orthogonality condition comes from a divergence at $k=0$ of the continuous basis of the primed Neumann matrices. Indeed, up to unimportant contributions, we have used the identification, see [34]

$$\delta_{nm} = \exp\left[(n-m)^2\left(\beta, \frac{1}{1+T'}\beta\right)\right] = \exp\left[-\frac{\ell^2}{b}(n-m)^2\left(\frac{(1-T')^2}{1+T'}\right)_{00}\right]. \tag{6.7}$$

Note that we don't really need to use different projectors on the worldvolume as the degeneracy is lifted by the different space-translations of the various projectors, however one can still use the $|\Lambda_n\rangle$'s in order to maintain the orthogonality as $\ell \to 0$.

Now we come to the spectrum.

Type (n,n) strings (the ones stretched between the same D-brane) are obtained by translation of strings on a single D24-brane

$$|(n,n); \{g\}, p_{\|}\rangle = e^{in\ell p_{\perp}}|\{g\}, p_{\|}\rangle^{(n)}, \tag{6.8}$$

where $|\{g\}, p_{\|}\rangle^{(n)}$ is an on-shell state of the previous section constructed on $|\Lambda_{nn}\rangle \otimes |\Xi'\rangle$. Thus we get N copies of the spectrum of a single D24-branes: N tachyons, N massless vectors etc... This gives a $\mathrm{U}(1)^N$ gauge symmetry.

The situation changes when we want to consider strings stretched between two different D-branes. In this case we expect that a shift in the mass formula is generated, proportional to the square of the distance between the two branes. In order to construct (i,j) states we have to translate the state $|\Xi'\rangle$ differently with respect its left/right degrees of freedom. We use the following identification for the left/right momentum

$$\hat{p} = \hat{p}_L + \hat{p}_R \tag{6.9}$$

$$\hat{p}_{L,R} = \frac{1}{\sqrt{b}}\left(\rho'_{L,R}a + \rho'_{L,R}a^{\dagger}\right)_0. \tag{6.10}$$

We then consider the state

$$e^{-in\ell\hat{p}_L - im\ell\hat{p}_R}|\Xi'\rangle \propto e^{(n\beta_L + m\beta_R)\cdot a'^{\dagger}}|\Xi'\rangle = |\Xi'_{nm}\rangle, \tag{6.11}$$

where we have defined

$$\beta_{L,R} = \rho'_{L,R}\beta. \tag{6.12}$$

The ρ' projectors obey the following properties up to midpoint subtleties, see later

$$\rho'_L + \rho'_R = 1 \tag{6.13}$$

$$(\rho'_{L,R})^2 = \rho'_{L,R} \tag{6.14}$$

$$\rho'_{L,R}\rho'_{R,L} = 0. \tag{6.15}$$

If we naively use these properties, using the formulas of [8], it is easy to prove that

$$|\Xi'_{nm}\rangle * |\Xi'_{pq}\rangle = e^{-\frac{1}{2}\left((nm+pq-nq)\beta\frac{1}{1-T'}\beta + (mp-nm-pq+nq)\beta\frac{1}{1-T'^2}\beta\right)}|\Xi'_{nq}\rangle \tag{6.16}$$

We can then normalize the above states in order to have

$$|\hat{\Xi}'_{nm}\rangle * |\hat{\Xi}'_{pq}\rangle = \delta_{mp}|\hat{\Xi}'_{nq}\rangle, \tag{6.17}$$

where

$$|\hat{\Xi}'_{nm}\rangle = \mathrm{e}^{\frac{1}{4}\beta\frac{n^2+m^2+2nmT'}{1-T'^2}\beta}\mathrm{e}^{(n\beta_L+m\beta_R)\cdot a'^\dagger}|\Xi'\rangle\,. \tag{6.18}$$

Note that this normalization is quite formal as the quantity $\beta\frac{1}{1+T}\beta$ is actually divergent, this is not however a real problem as open string states are not normalized by the LEOM's, moreover it should be noted that even if these left/right non-symmetric states have a vanishing normalization, they give rise to non vanishing objects (the projectors) by $*$ product.

Consider now, for simplicity, the "tachyon" state stretched from the i-th brane to the j-th. The corresponding state is given by

$$|(ij);p_\parallel\rangle = \mathcal{N}\mathrm{e}^{(-ta^\dagger+ix)p_\parallel}|\Xi\rangle\otimes|\hat{\Xi}'_{ij}\rangle\,. \tag{6.19}$$

Using (6.17) it is easy to see that the above state satisfies the LEOM

$$|(ij)\;;\;p_\parallel\rangle = |(ij)\;;\;p_\parallel\rangle*|N^{(\ell)}\rangle + |N^{(\ell)}\rangle*|(ij)\;;\;p_\parallel\rangle = 2^{-p_\parallel^2+1}|(nm)\;;\;p_\parallel\rangle \tag{6.20}$$

with $p_\parallel^2=1$, that is we don't get the usual mass shift proportional to the *distance*^{2} between the two D-branes.

However the algebra (6.17) is not quite correct. To elucidate this point it is worth considering the components of the level vector β in the continuous part of the diagonal basis of the primed Neumann matrices, see [33] for details.[6] We have

$$\beta(k) = -\frac{i\ell}{\sqrt{b}}(1+\mathrm{e}^{-\frac{\pi|k|}{2}})V_0(k)\,, \tag{6.21}$$

where $V_0(k)$ is the zero component of the normalized eigenvector of the continuous basis, [33]

$$V_0(k) = \sqrt{\frac{bk}{4\sinh\frac{\pi k}{2}}}\left[4+k^2\left(\Re F_c(k)-\frac{b}{4}\right)^2\right]^{-\frac{1}{2}}\,. \tag{6.22}$$

The β vector is finite at $k=0$,

$$\beta(0) = -\frac{i\ell}{\sqrt{2\pi}}\,, \tag{6.23}$$

hence its left/right decomposition is not well defined. This implies that it is not correct to consider the quantity

$$\gamma = \left(\beta_L,\frac{1}{1+T'}\beta_R\right) = -\frac{\ell^2}{b}\int_{-\infty}^{\infty}\theta(k)\theta(-k)\frac{\left(1+\mathrm{e}^{-\frac{\pi|k|}{2}}\right)^2}{1-\mathrm{e}^{-\frac{\pi|k|}{2}}}V_0(k)^2 \tag{6.24}$$

as vanishing since it is formally indeterminate (it is "$0\cdot\infty$"). Assuming that γ is non vanishing one easily obtains that the algebra (6.17) gets modified to

$$|\hat{\Xi}'_{nm}\rangle*|\hat{\Xi}'_{pq}\rangle = \delta_{mp}\,\mathrm{e}^{\frac{1}{4}\left[(n-p)^2+(m-q)^2\right]\gamma}\,|\hat{\Xi}'_{nq}\rangle\,. \tag{6.25}$$

[6]There are, of course, also the contributions from the discrete spectrum, but they are not singular for $0<b<\infty$

Taking this modification into account we obtain

$$|(nm)\ ;\ p_{\|}\rangle * |N\rangle + |N\rangle * |(nm)\ ;\ p_{\|}\rangle = 2^{-p_{\|}^2+1+\frac{1}{4}(n-m)^2\frac{\gamma}{\log 2}}|(nm)\ ;\ p_{\|}\rangle \tag{6.26}$$

that gives the mass formula

$$p_{\|}^2 = 1+\frac{1}{4}(n-m)^2\frac{\gamma}{\log 2}\,. \tag{6.27}$$

We recall that the mass for such a state should be given by ($\alpha' = 1$)

$$p_{\|}^2 = 1-\left(\frac{\Delta y_{nm}}{2\pi}\right)^2 = 1-\left(\frac{(n-m)\ell}{2\pi}\right)^2\,. \tag{6.28}$$

The two formulas agrees iff

$$\gamma = \left(\beta_L, \frac{1}{1+T'}\beta_R\right) = -\frac{\ell^2}{\pi^2}\log 2\,. \tag{6.29}$$

To verify this identity we need to regularize the ambiguous expression (6.24). We do it by substituting the lump Neumann coefficient T' with the wedge states one T'_N. We remind that, see [27]

$$T'_N = \frac{T'+(-T')^{N-1}}{1-(-T')^N} \tag{6.30}$$

$$T'_N \star T'_N = X' + (X'_+, X'_-)(1-\mathcal{T}'_N\mathcal{M}')^{-1}\mathcal{T}'_N\begin{pmatrix} X'_- \\ X'_+ \end{pmatrix} = T'_{2N-1}\,. \tag{6.31}$$

We have[7]

$$\gamma = \left(\beta, \rho_L(T')\frac{1}{1+T'\star T'}\rho_R(T')\beta\right) = \lim_{N\to\infty}\left(\beta\rho_L(T'_N)\frac{1}{1+T'_{2N}}\rho_R(T'_N)\beta\right), \tag{6.32}$$

where we have used the $*$-multiplication between wedge states

$$T'_N\star T'_N = T'_{2N-1} \backsimeq T'_{2N}\,, \qquad N \gg 1\,. \tag{6.33}$$

The matrices T'_N gets contributions from the continuous and the discrete spectrum but only the continuous spectrum is relevant in the large N limit, moreover it is only the region infinitesimally near the point $k = 0$ that really contributes. We have

$$\gamma = \left(\frac{i\ell}{\sqrt{2\pi}}\right)^2 \lim_{N\to\infty}\int_{-\infty}^{\infty} dk\rho_L^{(N)}(k)\frac{1}{1+t_{2N}(k)}\rho_R^{(N)}(k) \tag{6.34}$$

with

$$t_N(k) = \frac{-\mathrm{e}^{-\frac{\pi k}{2}}+\left(\mathrm{e}^{-\frac{\pi k}{2}}\right)^{N-1}}{1-\left(\mathrm{e}^{-\frac{\pi k}{2}}\right)^N} \tag{6.35}$$

$$\rho_L^{(N)}(k) = 1-\frac{1}{1+\left(\mathrm{e}^{-\frac{\pi k}{2}}\right)^{N-1}} \tag{6.36}$$

$$\rho_R^{(N)}(k) = \frac{1}{1+\left(\mathrm{e}^{-\frac{\pi k}{2}}\right)^{N-1}}\,, \tag{6.37}$$

[7]Note that the T' in the denominator of (6.24) is actually obtained by the projector equation $T'\star T' = T'$ that is violated in wedge-state regularization

where we have used the expression of $\rho'_{L,R}$ in terms of the sliver matrix and the $*$ Neumann coefficients, [9], and their (continuous) eigenvalues, [33]. Let's evaluate the integral in the large N limit ($x = -\frac{\pi k}{2}$, $y = Nx$)

$$\begin{aligned}
&\int_{-\infty}^{\infty} dk \rho_L^{(N)}(k) \frac{1}{1+t_{2N}(k)} \rho_R^{(N)}(k) \\
&\qquad = -\frac{2}{\pi} \int_{-\infty}^{\infty} dx \frac{\mathrm{e}^{Nx}\left(\mathrm{e}^{Nx}-1\right)}{\left(1-\mathrm{e}^{x}\right)\left(1+\mathrm{e}^{Nx}\right)\left(1+\mathrm{e}^{2Nx}\right)} + O\left(\frac{1}{N}\right) \\
&\qquad = -\frac{2}{N\pi} \int_{-\infty}^{\infty} dy \frac{\mathrm{e}^{y}\left(\mathrm{e}^{y}-1\right)}{\left(1-\mathrm{e}^{\frac{y}{N}}\right)\left(1+\mathrm{e}^{y}\right)\left(1+\mathrm{e}^{2y}\right)} + O\left(\frac{1}{N}\right) \\
&\qquad = \frac{2}{\pi} \int_{-\infty}^{\infty} \frac{dy}{y} \frac{\mathrm{e}^{y}\left(\mathrm{e}^{y}-1\right)}{\left(1+\mathrm{e}^{y}\right)\left(1+\mathrm{e}^{2y}\right)} + O\left(\frac{1}{N}\right) \\
&\qquad = \frac{2}{\pi} \log 2 + O\left(\frac{1}{N}\right)
\end{aligned} \tag{6.38}$$

so we get the right value of γ. Note that, when $N \to \infty$, the integrand completely localizes at $k = 0$, as claimed before.
As a last remark we would like to discuss about the position of the midpoint in the transverse direction for the states $|\Xi'_{nm}\rangle$. We know that for the usual lump solution we have, [19]

$$\hat{X}\left(\frac{\pi}{2}\right) |\Xi'\rangle = 0\,. \tag{6.39}$$

That is the lump functional has support on string states in which the midpoint is constrained to live on the worldvolume. This is interpreted as a Dirichlet condition, see also [35]. Moreover, since we have

$$\left[\hat{X}\left(\frac{\pi}{2}\right), p\right] = [x_0, p] = i\,, \tag{6.40}$$

it is immediate to see that

$$\hat{X}\left(\frac{\pi}{2}\right) \mathrm{e}^{-in\hat{p}\ell} |\Xi'\rangle = n\ell\, \mathrm{e}^{-in\hat{p}\ell} |\Xi'\rangle\,. \tag{6.41}$$

So shifted branes undergo a consistent change in boundary conditions.

The operator $\hat{X}\left(\frac{\pi}{2}\right)$ is proportional to the $k = 0$ position operator, [33]

$$\hat{X}\left(\frac{\pi}{2}\right) = 2\sqrt{\pi}\hat{x}_{k=0}\,. \tag{6.42}$$

It's easy to check that we have the following commutation relations

$$\left[2\sqrt{\pi}\hat{x}_k, p\right] = 2i\sqrt{\frac{2\pi}{b}} V_0(k) \tag{6.43}$$

$$\left[2\sqrt{\pi}\hat{x}_k, p_L\right] = 2i\sqrt{\frac{2\pi}{b}} V_0(k)\theta(-k) \tag{6.44}$$

$$\left[2\sqrt{\pi}\hat{x}_k, p_R\right] = 2i\sqrt{\frac{2\pi}{b}} V_0(k)\theta(k)\,. \tag{6.45}$$

That allow to write

$$\lim_{k\to 0^-} 2\sqrt{\pi}\hat{x}_k \mathrm{e}^{-i(n\hat{p}_L+m\hat{p}_R)\ell}|\Xi'\rangle = n\ell \mathrm{e}^{-i(n\hat{p}_L+m\hat{p}_R)\ell}|\Xi'\rangle \tag{6.46}$$

$$\lim_{k\to 0^+} 2\sqrt{\pi}\hat{x}_k \mathrm{e}^{-i(n\hat{p}_L+m\hat{p}_R)\ell}|\Xi'\rangle = m\ell \mathrm{e}^{-i(n\hat{p}_L+m\hat{p}_R)\ell}|\Xi'\rangle\,. \tag{6.47}$$

The string functional relative to this state is not continuous at the midpoint, this is the reason why the correct mass shell condition comes out from a twist anomaly. In the singular representation of VSFT in which the whole interior of a string is contracted to the midpoint, [20], these properties reproduce the expected change in the left/right boundary conditions, and show that the point $k = 0$ naturally accounts for D-branes moduli.

7. Conclusions

In this paper we have addressed the problem of how Chan-Paton degrees of freedom arise on the physical excitations of multiple D-branes. For coincident D-branes we have proved that these degrees of freedom can be encoded in the algebra of Laguerre polynomials, discovered in [22], we expect however that other (gauge equivalent) descriptions can be given. We have further shown that if we consider an array of parallel displaced D24-branes, the expected higgsing $\mathrm{U}(N) \to \mathrm{U}(1)^N$ takes place. The shift in the mass formula is due to a twist anomaly, a contribution which is completely localized at the point $k = 0$ of the continuous spectrum of the Neumann matrices with zero modes. This is the same phenomenon, discovered in [23], that gives rise to the correct mass formula for open string states on a single D25-brane. This result confirms once more that observables in VSFT are associated to midpoint subtleties. This fact is hardly a surprise given that the field redefinition relating OSFT on the tachyon vacuum to VSFT completely shrinks the body of a string to its midpoint and "resolves" the endpoints into the left/right halves, [20]. This, we believe, is the reason why Chan-Paton factors are to be found in left/right excitations of VSFT classical solutions, and not in the endpoints, see [31].

It would be interesting to give a BCFT description of our construction. In particular the properties (6.46), (6.47) suggest that stretched states should be given by the insertion on the boundary of the lump surface state of a boundary changing vertex operator, in a way similar to [10], but with one insertion more . Another interesting development would be to switch on interactions in order to explicitly realize the $\mathrm{U}(N)$ dynamical symmetry.

We hope we have given a clear picture on how some topics related to background independence naturally emerge from the string field algebra. We believe this to be relevant in understanding the elusive nature of closed string states around the tachyon vacuum.

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A. Proof of (4.13)

A general open string state on the sliver $|\Xi\rangle$ can be obtained differentiating the generating state, see [16]

$$|\phi_\beta\rangle = \mathrm{e}^{-(tp+\beta)\cdot a^\dagger}|\Xi\rangle \mathrm{e}^{ipx}\,, \tag{A.1}$$

where

$$t = 3\frac{T^2-T+1}{1+T}v_0 \tag{A.2}$$

is the on-shell tachyon vector, [13] and β_μ is a level-Lorentz vector.

The $|\Lambda_n\rangle$'s can be generated by the state ,[22]

$$|\Xi_\lambda\rangle = \mathrm{e}^{\lambda\cdot a^\dagger}|\Xi\rangle\,. \tag{A.3}$$

General formulas of [9] and [16] allows to compute

$$|\phi_\beta\rangle * |\Xi_\lambda\rangle = \mathrm{e}^{-Gp^2+A_{LR}(\beta,\lambda)}\mathrm{e}^{-(tp+\rho_L\beta-\rho_R\lambda)\cdot a^\dagger}|\Xi\rangle\mathrm{e}^{ipx} \tag{A.4}$$

$$|\Xi_\lambda\rangle * |\phi_\beta\rangle = \mathrm{e}^{-Gp^2+A_{RL}(\beta,\lambda)}\mathrm{e}^{-(tp+\rho_R\beta-\rho_L\lambda)\cdot a^\dagger}|\Xi\rangle\mathrm{e}^{ipx}\,, \tag{A.5}$$

where

$$\begin{aligned} A_{LR}(\beta,\lambda) = &-\frac{1}{2}\left(\beta\cdot,\frac{T}{1-T^2}\beta\right)+\left(\beta\cdot,\frac{\rho_R-T\rho_L}{1-T^2}C\lambda\right)-\frac{1}{2}\left(\lambda\cdot,\frac{T}{1-T^2}\lambda\right)+\\ &+p\cdot\left(\left(t,\frac{T}{1-T^2}\beta\right)-\left(t,\frac{\rho_L-T\rho_R}{1-T^2}\lambda\right)+\left(t,\frac{\rho_R+T\rho_L}{1-T^2}\beta\right)-\left(t,\frac{\rho_L-\rho_R}{1-T^2}\lambda\right)\right)\end{aligned}$$

and

$$A_{RL}(\beta,\lambda) = A_{LR}(\beta,\lambda)\Big|_{\rho_L\to\rho_R,\ \rho_R\to\rho_L}\,. \tag{A.6}$$

We can restrict the polarization β_n^μ to the $k=0$ component, indeed every physical excitation of the tachyon wave function $\mathrm{e}^{-tp\cdot a^\dagger+ipx}|\Xi\rangle$ should be localized there, see [14, 16].

Therefore is not restrictive to ask

$$(\beta\cdot, f(T)\xi) = 0 \tag{A.7}$$

once the half string vector ξ^μ vanishes rapidly enough at $k=0$.

Moreover asking for (4.14) to be satisfied, using (3.12), it is easy to show that

$$|\{g\},p\rangle * |N\rangle + |N\rangle * |\{g\},p\rangle = |\{g\},p\rangle * |\Xi\rangle + |\Xi\rangle * |\{g\},p\rangle\,, \tag{A.8}$$

as claimed.

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